

**MATH 464 (THEORY OF PROBABILITY)  
HOMEWORK 8**

FALL 2017

**Due on: Thursday 10-26-2017.**

- (1) For any  $\omega > 0$ , let

$$\Gamma(\omega) := \int_0^{\infty} x^{\omega-1} e^{-x} dx.$$

Show that if  $\omega$  is a positive integer then  $\Gamma(\omega) = (\omega - 1)!$

- (2) Find the mean and variance of the Gamma( $\lambda, \omega$ ) distribution.
- (3) Suppose  $X$  is an exponential random variable with parameter  $\lambda = 1$ . Find the distribution and density functions of  $Y = \ln(X)$ .  
**Note:** This is called the double exponential distribution.
- (4) Suppose  $X \sim N(0, 1)$ . Use integration by parts to show that  $\mathbb{E}(X^k) = (k - 1)\mathbb{E}(X^{k-2})$ . Derive that  $\mathbb{E}(X^k) = 0$  for all odd  $k \geq 1$ . Compute  $\mathbb{E}(X^4)$  and  $\mathbb{E}(X^6)$ . Derive a general formula for  $\mathbb{E}(X^{2k})$ .